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# *At least* as *n*-ary disjunction: Scales, context, and exhaustification

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## 1 Introduction

A hallmark feature of the scalar focus operator *at least* is its capacity to convey speaker ignorance (Kay 1992, Krifka 1999, Geurts & Nouwen 2007):

(1) a. LeBron scored at least [20 points]$_F$ in last night's game.
b. Grover ate at least [some]$_F$ of his dinner.
c. Mabel won a [silver]$_F$ medal, at least.

Focus placement has a constraining effect on the scale that *at least* operates over, as well as the ignorance inferences that result

(2) a. At the very least, LeBron scored [20 points]$_F$ in last night's game.
b. At the very least, [LeBron]$_F$ scored 20 points in last night's game.
c. At the very least, LeBron scored 20 points in [last night's]$_F$ game.

- semantic approaches: ignorance inferences form part of truth-conditional meaning (Geurts & Nouwen 2007, Nouwen 2010)
- pragmatic approaches: ignorance inferences are conversational implicatures arising from the interaction of *at least*'s basic semantic properties with general pragmatic principles (e.g., Büring 2008, Cummins & Katsos 2010, Coppock & Brockhagen 2013, Mayr 2013, Schwarz 2013, 2016, Kennedy 2015, Nouwen 2015; see also Cohen & Krifka 2014 for an account in terms of meta-speech acts)

A near-universal impulse of pragmatic approaches is to draw an analogy to ordinary disjunction, which also conveys speaker ignorance:

(3) Grover ate tuna, chicken, or duck for dinner.

Capitalizing on this analogy has proven to be surprisingly difficult:

- a simple view: *at least* expresses *n*-ary disjunction over its associated scalar item and all higher ones
- this simple view adequately captures the truth-conditional contributions of *at least*, but appears to mischaracterize its pragmatic effects

**Goal for today's talk:** to show how the problems for this simple view may be overcome, once closer attention is paid to the scales that *at least* operates over. In particular, these scales:

(i) may be fundamentally pragmatic or contextual in nature (vs. purely logical or otherwise conventional), and

(ii) are never ordered by logical entailment.

## 2 *At least* and disjunction: An elusive analogy

### *2.1* At least *as a scalar focus operator*

Simplifying assumption: in the input to semantic interpretation, *at least* attaches to a proposition-denoting constituent, some portion of which bears F-marking.

(4) at least [LeBron scored [20 points]$_F$ in last night's game]

- the semantic effect of F-marking, $[\![\cdot]\!]_f$, is to evoke a set of alternative semantic values, or focus alternatives (Rooth 1985, Fox & Katzir 2011)
- *at least* presupposes that these alternatives form partially-ordered scales (compare to *only*, *even* and other scalar focus operators)

(5) a. ⟦ LeBron scored [20 points]$_F$ in last night's game ⟧$_f$
= { … ⟦ LeBron scored 22 points in last night's game ⟧ >
⟦ LeBron scored 21 points in last night's game ⟧ >
⟦ LeBron scored 20 points in last night's game ⟧ >
⟦ LeBron scored 19 points in last night's game ⟧ >
⟦ LeBron scored 18 points in last night's game ⟧ > … }

b. ⟦ Grover ate [some]$_F$ of his dinner ⟧$_f$ = { ⟦ Grover ate all of his dinner ⟧ >
⟦ Grover ate most of his dinner ⟧ >
⟦ Grover ate some of his dinner ⟧ }

c. ⟦ Mabel won a [silver]$_F$ medal ⟧$_f$ = { ⟦ Mabel won a gold medal ⟧ >
⟦ Mabel won a silver medal ⟧ >
⟦ Mabel won a bronze medal ⟧ }

### *2.2* At least *as* n*-ary disjunction: Truth-conditional vacuity and weakening*

With entailment scales, *at least* appears to be truth-conditionally vacuous:

(6) a. Grover ate some of his dinner.
b. Grover ate at least [some]$_F$ of his dinner.

- (6a,b) compatible with G's eating most/all of dinner

With non-entailment scales, its presence has a weakening effect:

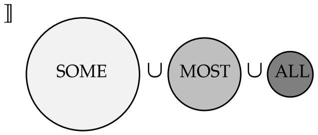
(7) a. Mabel won a silver medal.
b. Mabel won a [silver]$_F$ medal, at least.

- only (7b) compatible with M's winning gold

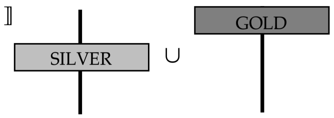
Krifka (1999): *at least* creates an *n*-ary disjunction over its prejacent, or scope, and all higher focus alternatives.

- when the alternatives are ordered by logical entailment, the entire disjunction is truth-conditionally equivalent to its weakest disjunct, i.e., the prejacent itself

(8) ⟦ at least [Grover ate [some]$_F$ of his dinner] ⟧
= ⟦ Grover ate some of his dinner ⟧ ∨
⟦ Grover ate most of his dinner ⟧ ∨
⟦ Grover ate all of his dinner ⟧
≡ ⟦ Grover ate some of his dinner ⟧

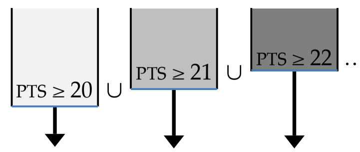


(9) ⟦ at least [Mabel won a [silver]$_F$ medal] ⟧
= ⟦ Mabel won a silver medal ⟧ ∨
⟦ Mabel won a gold medal ⟧

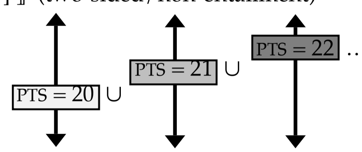



One-sided/entailment (Horn 1972) vs. two-sided/non-entailment (Horn 1992, Geurts 2006, Kennedy 2013) semantics for bare numerals:

(10) a. ⟦ at least [LeBron scored [20 points]$_F$ ] ⟧ (one-sided/entailment)
= ⟦ LeBron scored 20 points ⟧ ∨
⟦ LeBron scored 21 points ⟧ ∨
⟦ LeBron scored 22 points ⟧ ∨ …
≡ ⟦ LeBron scored 20 points ⟧




b. ⟦ at least [LeBron scored [20 points]$_F$ ] ⟧ (two-sided/non-entailment)
= ⟦ LeBron scored 20 points ⟧ ∨
⟦ LeBron scored 21 points ⟧ ∨
⟦ LeBron scored 22 points ⟧ ∨ …




(11) ⟦ at least S ⟧ = $\lambda w.\exists q[\, q \in \{ p \mid p \in ⟦ S ⟧_f \; \& \; p \geq ⟦ S ⟧ \} \; \& \; q(w) \,]$

*2.3 At least as n-ary disjunction: Symmetric competitors and ignorance inferences*

Neo-Gricean approach to implicature calculation (Grice 1967/1975, 1978, Horn 1972): listeners draw additional inferences about the speaker's communicative intent in uttering some sentence *S* by reasoning about the members of a formally-defined set of pragmatic competitors to *S*, COMP(*S*).

The Standard Recipe for deriving Quantity implicatures within the neo-Gricean approach (Sauerland 2004, Fox 2007a, Geurts 2011):

(i) Listeners infer from a speaker's utterance of *S* that the speaker is not certain of the truth of any logically stronger competitor *S′* belonging to COMP(*S*):

¬BEL(*S′*) (Weak Quantity Implicature)

(ii) Listeners may additionally take the speaker to be competent with respect to the stronger competitor *S′* :

BEL(*S′*) ∨ BEL(¬*S′*) (Competence Assumption)

(iii) Listeners may thus come to infer that the speaker is certain that *S′* is false:

BEL(¬*S′*) (Strong Quantity Implicature)



A compelling feature of the Standard Recipe is the unified account that it provides of upper-bounding <u>and</u> ignorance inferences as subspecies of Quantity implicatures:

(12) Grover had tuna or chicken for dinner.

COMP(12) = { <u>T ∨ C</u> , T , C , T & C }

(i) BEL(T ∨ C) (Quality)
(ii) ¬BEL(T & C), ¬BEL(T), ¬BEL(C) (Weak Quantity)
(iii) BEL(T & C) ∨ BEL(¬(T & C)) (Competence re: T & C)
(iv) BEL(¬(T & C)) (Strong Quantity/Upper-bounding)

- what happens if we assume competence re: one of the disjuncts, e.g., T ?

(v) BEL(T) ∨ BEL(¬T) (Competence re: T)
(vi) BEL(¬T) (Strong Quantity)

- (i) and (vi) together entail BEL(C), which contradicts (ii)!

(vii) ¬(BEL(T) ∨ BEL(¬T)) (Non-Competence/Ignorance re: T)
≡ ¬BEL(T) & ¬BEL(¬T)
≡ POSS(T) & POSS(¬T)
(viii) POSS(C) & POSS(¬C) (Ignorance re: C via analogous reasoning)

- the individual disjuncts T, C form a pair of **symmetric stronger competitors**: strengthening the first's weak Quantity implicature under Competence contradicts the second's weak Quantity implicature, and vice versa

- the Standard Recipe derives ignorance implicatures for any pair of symmetric competitors (Fox 2007a, Schwarz 2013, 2016)

Generalizing from binary to *n*-ary disjunction: COMP contains each individual disjunct and is closed under disjunction, yielding a full set of stronger quantificational-domain competitors (Alonso-Ovalle 2006, Chierchia 2013).

(13) Grover ate tuna, chicken, or duck for dinner.

COMP(13) = { <u>T ∨ C ∨ D</u> ,
T ∨ C , D , symmetric
T ∨ D , C , symmetric
C ∨ D , T } symmetric
= { $\lambda w.\exists p[\, p \in Q \;\&\; p(w) \,] \mid Q \subseteq \{$ T , C , D $\}$ }

POSS(T) & POSS(¬T)
POSS(C) & POSS(¬C) (Ignorance re: each individual disjunct)
POSS(D) & POSS(¬D)

- reasoning about stronger quantificational-domain competitors has also been called upon to account for the pragmatic behavior of epistemic indefinites like German *irgendein* (Kratzer & Shimoyama 2002), Spanish *algún* (Alonso-Ovalle & Menendez-Benito 2010), and Romanian *vreun* (Falaus 2014)

The simple(st?) extension to *at least*: COMP contains the prejacent and all higher focus alternatives (the individual "disjuncts"), and is closed under disjunction, again yielding a full set of stronger quantificational-domain competitors.

(14) at least [LeBron scored [20 points]$_F$] (two-sided / non-entailment)

COMP(14) = { $\underline{\text{PTS} = 20 \vee \text{PTS} = 21 \vee \text{PTS} = 22 \vee \text{PTS} = 23 \vee \ldots}$ ,
≡ PTS ≥ 20

PTS = 21 ∨ PTS = 22 ∨ PTS = 23 ∨ … , PTS = 20 , symmetric
≡ PTS ≥ 21

PTS = 20 ∨ PTS = 22 ∨ PTS = 23 ∨ … , PTS = 21 , symmetric
≡ PTS = 20 ∨ PTS ≥ 22

PTS = 20 ∨ PTS = 21 ∨ PTS = 23 ∨ … , PTS = 22 , symmetric
≡ PTS = 20 ∨ PTS = 21 ∨ PTS ≥ 23

… }

$= \{ \lambda w.\exists p[\, p \in Q \,\&\, p(w) \,] \mid Q \subseteq \{ \text{PTS} = n \mid n \geq 20 \} \}$

POSS(PTS = 20) & POSS(¬PTS = 20)
POSS(PTS = 21) & POSS(¬PTS = 21)
POSS(PTS = 22) & POSS(¬PTS = 22)
…
________
$\forall n[\, n \geq 20 \rightarrow (\text{POSS}(\text{PTS} = n) \,\&\, \text{POSS}(\neg\text{PTS} = n)) \,]$
(Ignorance re: prejacent and all higher focus alternatives)

*2.4 At least as n-ary disjunction: Too much ignorance and "unsuspended" upper bounds*

**Problem:** unlike ordinary *n*-ary disjunction, *at least* does not generally convey total ignorance regarding each of its "disjuncts" (Schwarz 2013, 2016, Alexandropoulou et al. 2015, Ander-Mendia 2015, Nouwen 2015).

(15) Grover ate tuna, chicken, or duck for dinner, though…
#...I'm sure that he didn't eat tuna.
#...I'm sure that he didn't eat chicken.
#...I'm sure that he didn't eat duck.

- implausible or impossible focus alternatives need not be considered possible

(16) LeBron scored at least 20 points in last night's game.
(likely will not implicate 'possibly 350 points')

- privileged status of ignorance inference regarding the prejacent vs. those regarding individual higher alternatives

(17) Grover ate at least some of his dinner, though…
#...I'm sure that he didn't eat (just) some of it.
…I'm sure that he didn't eat all of it.
#...I'm sure that he didn't eat more than (just) some of it.



(18) At the very least, he is a colonel, though…
#...I'm sure that he is not a colonel.
…I'm sure that he is not a general.
#...I'm sure that he is nothing higher than a colonel.

**Observation:** *at least* reliably conveys ignorance about its prejacent and the disjunction of all higher focus alternatives.

(19) at least [LeBron scored [20 points]$_F$] (two-sided / non-entailment)

POSS(PTS = 20) & POSS(¬PTS = 20) (Ignorance re: prejacent)
POSS(PTS ≥ 21) & POSS(¬PTS ≥ 21) (Ignorance re: disjunction of all higher focus alternatives)

A possible solution: stipulate that the competitor set evoked by *at least* is more impoverished, and thus exhibits less symmetry, than the set evoked by ordinary disjunction (Büring 2008, Schwarz 2013, 2016, Kennedy 2015, Nouwen 2015).

COMP(19) = { $\underline{\text{PTS} \geq 20}$ ,
PTS ≥ 21 , PTS = 20 , symmetric
(…) }

Another possible solution: locate the difference between *at least* and ordinary disjunction not in their respective competitor sets, but rather in the determination of their respective quantificational domains (see §3 below).

**Problem #2:** *at least*'s capacity to suspend upper-bounding inferences when it operates over entailment scales goes unaccounted for.

- *at least* is one of Horn's (1972) implicature-suspension devices: by virtue of conveying ignorance regarding stronger competitors, it calls off the strong Quantity implicatures that would typically arise in its absence

(20) a. Grover ate some of his dinner.
b. Grover ate at least [some]$_F$ of his dinner.

- somewhat perversely, the hypothesized competitor sets for (20a) and (20b) turn out to be equivalent, guaranteeing the Standard Recipe cannot distinguish between them

COMP(20a) = { $\underline{\text{SOME}}$, MOST , ALL }

COMP(20b) = { $\underline{\text{SOME} \vee \text{MOST} \vee \text{ALL}}$ ,
≡ SOME

SOME ∨ MOST , ALL , not symmetric !!!
≡ SOME

SOME ∨ ALL , MOST , not symmetric !!!
≡ SOME

MOST ∨ ALL , SOME } not symmetric !!!
≡ MOST

= { $\underline{\text{SOME}}$, MOST , ALL }

**Observation:** the same logical property that underlies the truth-conditional vacuity of *at least* in (20b) (A ∨ B ≡ A whenever B entails A) prevents the correct derivation of ignorance inferences. Instead, (20b) is predicted to convey the same upper-bounding inferences as (20a).

BEL(¬MOST), BEL(¬ALL) (Strong Quantity / Upper-bounding)

This problem of implicature "unsuspension" arises for any scale ordered by entailment, e.g., bare numerals under their one-sided semantics (compare to (14)):

(21) at least [LeBron scored [20 points]$_F$] (one-sided / entailment)

COMP(21) = { $\underline{\text{PTS} \geq 20 \vee \text{PTS} \geq 21 \vee \text{PTS} \geq 22 \vee \text{PTS} \geq 23 \vee \ldots}$ ,
≡ PTS ≥ 20

PTS ≥ 21 ∨ PTS ≥ 22 ∨ PTS ≥ 23 ∨ … , PTS ≥ 20 , not symmetric !!!
≡ PTS ≥ 21

PTS ≥ 20 ∨ PTS ≥ 22 ∨ PTS ≥ 23 ∨ … , PTS ≥ 21 , not symmetric !!!
≡ PTS ≥ 20

PTS ≥ 20 ∨ PTS ≥ 21 ∨ PTS ≥ 23 ∨ … , PTS ≥ 22 , not symmetric !!!
≡ PTS ≥ 20

… }
= { $\underline{\text{PTS} \geq 20}$ , PTS ≥ 21 , PTS ≥ 22 , … }

BEL(¬PTS ≥ 21), BEL(¬PTS ≥ 22), … (Strong Quantity / Upper-bounding)

**Summary:** the *n*-ary disjunction view of *at least* correctly characterizes its truth-conditional contributions, but not its pragmatic effects: in some cases, it predicts too many ignorance inferences, while for entailment scales, it predicts none at all.

## 3 Getting (just) enough ignorance

### *3.1 Scales, contextual relevance, and contextual indeterminacy*

Even more diversity in the scales that *at least* may operate over:

- non-conventional, contextually-defined scales (Kay 1992)

(22) (Regarding a trip from San Francisco to New York vs. the return trip)
At the very least, he's made it to [Chicago]$_F$ by now.

(23) (Simplified rules for craps: if a player throws a 2, 3, or 12 on her first roll, she loses. If a player throws a 7 or 11 on her first roll, she wins. If a player throws a 4, 5, 6, 8, 9, or 10 on her first roll, she gets subsequent rolls / chances to win.)
(Upon seeing a player collect her winnings)
She at least threw [a 4, 5, 6, 8, 9, or 10]$_F$ on her first roll.

- contextual restrictions on conventionally-defined scales

(24) A: Does Pete like any of Nora's relatives?
B: He at least likes [her father]$_F$.
(may implicate 'and possibly her mother too')
(will not implicate 'and possibly Barack Obama too')



Parallel behavior to other scalar focus operators, e.g., *even* and *only*:

(25) A: Does Fred eat sushi?
B: Of course he does! He even eats [squid]$_F$.

(26) John brought Tom, Bill, and Harry to the party, but
he only introduced [Bill]$_F$ to Sue. (Rooth 1996)

Rooth (1992, 1996): scalar focus operators are anaphoric to a contexually-provided scale *C*, the identity of which is constrained / indicated by focus placement.

(27) ⟦ at least$_C$ S ⟧ = $\lambda w.\exists q[\, q \in \{\, p \mid p \in \underline{C} \;\&\; p \geq ⟦ \text{S} ⟧ \,\} \;\&\; q(w)\,]$
Presuppositional constraints on *C*:
(i) $C \subseteq ⟦ \text{S} ⟧_f$ (partial recoverability of the scale *C* from F-marking)
(ii) $⟦ \text{S} ⟧ \in C$ (speaker's choosing to evoke focus alternatives with prejacent guarantees the prejacent's contextual relevance)
(iii) $|C| > 1$ (non-triviality of the scale *C*)

**Claim**: COMP contains the prejacent and all <u>contextually-relevant</u> higher focus alternatives and is closed under disjunction, yielding a full set of stronger quantificational-domain competitors <u>relative to *C*</u>.

(28) Let $C = \{\, \text{PTS} = n \mid 10 \leq n < 100 \,\}$. Then,
⟦ at least$_C$ [LeBron scored [20 points]$_F$] ⟧ (two-sided / non-entailment)
= PTS = 20 ∨ PTS = 21 ∨ PTS = 22 ∨ … ∨ PTS = 99
≡ 20 ≤ PTS < 100

COMP(28) = { $\underline{\text{PTS} = 20 \vee \text{PTS} = 21 \vee \text{PTS} = 22 \vee \ldots \vee \text{PTS} = 99}$ ,
≡ 20 ≤ PTS < 100

PTS = 21 ∨ PTS = 22 ∨ … ∨ PTS = 99 , PTS = 20 , symmetric
≡ 21 ≤ PTS < 100

PTS = 20 ∨ PTS = 22 ∨ … ∨ PTS = 99 , PTS = 21 , symmetric
≡ PTS = 20 ∨ 22 ≤ PTS < 100

…

PTS = 20 ∨ PTS = 21 ∨ … ∨ PTS = 98 , PTS = 99 , symmetric
≡ 20 ≤ PTS < 99

… }

POSS(PTS = 20) & POSS(¬PTS = 20)
POSS(PTS = 21) & POSS(¬PTS = 21)
…
POSS(PTS = 99) & POSS(¬PTS = 99)
———
$\forall n[\, 20 \leq n < 100 \rightarrow$ (POSS(PTS = $n$) & POSS(¬PTS = $n$)) ]
(Ignorance re: prejacent and all contextually relevant higher alternatives)

- the lack of ignorance inferences regarding implausible / impossible alternatives follows so long as such alternatives are necessarily irrelevant

Contextual indeterminacy and privileged prejacents: what is the most that a listener can infer when s/he does not know the speaker's intended value for the scale *C*?

*C* = { PTS = 19 , PTS = 20 , PTS = 21 }

POSS(PTS = 20) & POSS(¬PTS = 20)
POSS(PTS = 21) & POSS(¬PTS = 21) — entails POSS(PTS ≥ 21)

*C* = { PTS = 18 , PTS = 20 , PTS = 22 }

POSS(PTS = 20) & POSS(¬PTS = 20)
POSS(PTS = 22) & POSS(¬PTS = 22) — entails POSS(PTS ≥ 21)

*C* = { PTS = 10 , PTS = 20 , PTS = 30 , PTS = 40 , PTS = 50 }

POSS(PTS = 20) & POSS(¬PTS = 20)
POSS(PTS = 30) & POSS(¬PTS = 30) — entails POSS(PTS ≥ 21)
POSS(PTS = 40) & POSS(¬PTS = 40) — entails POSS(PTS ≥ 21)
POSS(PTS = 50) & POSS(¬PTS = 50) — entails POSS(PTS ≥ 21)

- since the prejacent PTS = 20 is presupposed to belong to *C*, every admissible value for *C* will yield an ignorance inference about the prejacent
- no individual higher focus alternative is guaranteed to belong to *C*, but for the inclusion of *at least* is non-trivial, <u>some</u> higher alternative must belong to *C*
- in the extreme case, the most that a listener can infer is what follows under <u>any</u> admissible value for *C*:

POSS(PTS = 20) & POSS(¬PTS = 20) (Ignorance re: prejacent)
POSS(PTS ≥ 21) & POSS(¬PTS ≥ 21) (Ignorance re: disjunction of all higher focus alternatives)

*3.2 Embedded exhaustification and implicature re-suspension*

The unsuspension problem arises for another of Horn's implicature-suspension devices, namely ordinary disjunction (Chierchia et al. 2009, 2011):

(29) a. Grover ate some or (even) most of his dinner.
b. Grover ate [[tuna or chicken] or both] for dinner.
c. LeBron scored 20 or 21 points in last night's game.

COMP(29a) = { <u>SOME ∨ MOST</u> (≡ SOME) , SOME , MOST , <u>SOME & MOST</u> (≡ MOST) }
= { <u>SOME</u> , MOST } — not symmetric !!!

BEL(¬MOST) — (Strong Quantity / Upper-bounding)

Chierchia et al.'s (2009, 2011) solution: the disjunctions in (29) are parsed with a covert scalar focus operator, *exh*, in the first disjunct.

(30) [*exh* [Grover ate [some]$_F$ of his dinner]] or [Grover ate most of his dinner]

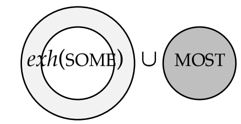


- *exh* strengthens the meaning of its prejacent by conjoining it with the negations of all logically stronger focus alternatives, resulting in an upper-bounded truth-conditional meaning (see Fox 2007 for a more refined version)

(31) $[\![\, exh\ S \,]\!] = \lambda w.\ [\![\, S \,]\!](w)\ \&\ \forall q[\ q \in \{\, p \mid p \in [\![\, S \,]\!]_f\ \&\ p(w)\,\} \rightarrow [\![\, S \,]\!] \subseteq q\ ]$

- the presence of *exh* disrupts any entailment between the disjuncts—post-exhaustification, the two disjuncts are in fact logically incompatible

(32) ⟦ [*exh* [G. ate [some]$_F$ of his dinner]] or [G. ate most of his dinner] ⟧
= *exh*(SOME) ∨ MOST
≡ (SOME & ¬MOST & ¬ALL) ∨ MOST
≡ (SOME & ¬MOST) ∨ MOST
≡ SOME




- the presence of *exh* also results in symmetric competitors, thus re-suspending the upper-bounding inferences in favor of ignorance inferences

COMP(30) = { <u>*exh*(SOME) ∨ MOST</u> (≡ SOME) ,
*exh*(SOME) , MOST , — symmetric
*exh*(SOME) & MOST (≡ ⊥) }
= { <u>SOME</u> , *exh*(SOME) , MOST }

POSS(*exh*(SOME)) & POSS(¬*exh*(SOME))
POSS(MOST) & POSS(¬MOST) — (Ignorance re: each individual disjunct)

(NB: Chierchia et al. 2009, 2011 introduce *exh* in the course of developing their Grammatical Theory of scalar implicatures, a direct competitor to the Neo-Gricean Theory. I continue to assume the latter for the purposes of implicature calculation.)

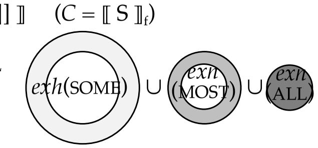
**Claim:** *at least* never operates over entailment scales.

- in all such putatives instances, *exh* occurs somewhere in the prejacent and associates with the same F-marked constituent as does *at least* (see Krifka 1991, Wold 1996 on the compositional interpretation of such constructions; see also Crnic 2012, Kilbourn-Ceron 2016 for other cases of embedded *exh*)

(33) at least$_C$ [*exh* [Grover ate [some]$_F$ of his dinner]]

- exhaustification of the prejacent disrupts any entailments amongst the focus alternatives without affecting the overall truth-conditional meaning

(34) ⟦ at least$_C$ [$_S$ *exh* [Grover ate [some]$_F$ of his dinner]] ⟧ (C = ⟦ S ⟧$_f$)
= *exh*(SOME) ∨ *exh*(MOST) ∨ *exh*(ALL)
≡ (SOME & ¬MOST & ¬ALL) ∨ (MOST & ¬ALL) ∨ ALL
≡ (SOME & ¬MOST) ∨ (MOST & ¬ALL) ∨ ALL
≡ SOME



- the presence of *exh* again results in symmetric competitors, solving the implicature unsuspension problem

COMP(33) = { $\underline{exh(\textsc{some}) \vee exh(\textsc{most}) \vee exh(\textsc{all})}$ ,
≡ SOME

$exh(\textsc{most}) \vee exh(\textsc{all})$ , $exh(\textsc{some})$ , symmetric
≡ MOST

$exh(\textsc{some}) \vee exh(\textsc{all})$ , $exh(\textsc{most})$ , symmetric

$exh(\textsc{some}) \vee exh(\textsc{most})$ , $exh(\textsc{all})$ } symmetric
≡ SOME & ¬ ALL

POSS(*exh*(SOME)) & POSS(¬*exh*(SOME))
POSS(*exh*(MOST)) & POSS(¬*exh*(MOST))
POSS(*exh*(ALL)) & POSS(¬*exh*(ALL))
(Ignorance re: prejacent and all (contextually relevant) higher alternatives)

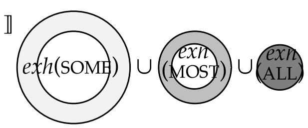
## 4 *At least*, *exh*, and truth-conditional (non-)vacuity

**Question:** what motivates the presence of *exh* when *at least* (apparently) operates over entailment scales?

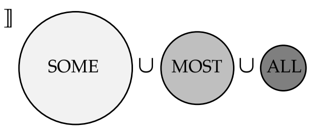
**Claim:** *at least* is subject to a ban on locally vacuous semantic composition (see Katzir & Singh 2013 on *or*, see also Meyer 2015).

(35) [ at least$_C$ S ] is deviant if ⟦ at least$_C$ S ⟧ ≡ ⟦ S ⟧ .

(36) ⟦ at least [$_S$ *exh* [Grover ate [some]$_F$ of his dinner]] ⟧
= $exh(\textsc{some}) \vee exh(\textsc{most}) \vee exh(\textsc{all})$
≡ SOME
≠ ⟦*exh* [Grover ate [some]$_F$ of his dinner]⟧




(37) ⟦ at least [Grover ate [some]$_F$ of his dinner] ⟧
= SOME ∨ MOST ∨ ALL
≡ SOME
= ⟦Grover ate [some]$_F$ of his dinner⟧




**Prediction:** *at least* should be deviant whenever *exh* is itself vacuous, and fails to strengthen its prejacent's meaning (⟦ *exh* S ⟧ ≡ ⟦ S ⟧).

- *more than n* comparative numerals (Fox & Hackl 2006, Spector 2014)

(38) A: How many points did LeBron score?
B: #I don't know exactly, but he at least scored [more than 20]$_F$ .

- *W or S* disjunctions

(39) #At the very least, [[Nora or Mabel] or both]$_F$ will be there.



- *at least* ! (and other implicature-suspension devices?)

(40) #At the very least, he scored [at least 20 points]$_F$ .

In each case, an intervening necessity modal (*at least* ≻ *exh* ≻ □ ≻ $[\alpha]_F$) allows for non-vacuous exhaustification (Fox & Hackl 2006, Fox 2007b), and thus restores felicity:

(41) A: How many points does LeBron need to score (to win the scoring title)?
B: I don't know exactly, but he at least needs to score [more than 20]$_F$ .

(42) At the very least, you need to talk to [[Nora or Mabel] or both]$_F$ .

(43) At the very least, he needs to score [at least 20 points]$_F$ .

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