

Epistemic vs. Concessive *At Least*: The Least *At Least* Can Do (While Still Managing To Do At Least Something)

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1 Introduction

Kay (1992) distinguishes epistemic (1a) and evaluative/concessive (1b) readings for the English scalar operator *at least*: whereas (1a) conveys that the speaker considers the higher scalar values *most*, *all* to be possible, (1b) instead conveys that these higher scalar values would have been preferable.

- (1) a. Paws ate **at least** [some]_F of his food. (term-level, epistemic)
b. **At least** Paws ate [some]_F of his food. (sentence-initial, concessive)

- there is a large (and ever-growing!) body of work devoted to the epistemic reading of *at least*, which has converged on the view that these inferences are pragmatic in nature (see references in §2.1)
- comparatively little work has explored the concessive reading of *at least*, with little consensus regarding its truth-conditional import and pragmatic effects (see Nakanishi & Rullmann 2009a,b, Gast 2011, Grosz 2011a,b, Biezma 2013)

Nakanishi & Rullmann (2009a) and Grosz (2011a,b) document that in many languages, the same expression may express scalar uncertainty or preference:

- (2) French *au moins* (Kay 1992)
- a. Ça vaut **au moins** 30 F. (epistemic)
That costs at-least 30 francs
'That costs at least [30 francs]_F.'
- b. J'aime bien ce restaurant. (concessive)
I like well that restaurant
au moins, on sait ce que l'on y mange
at-least one knows that which one there eats
'I like that restaurant: at least you know what you're eating there.'

- cf. also Dutch *tenminste*, Spanish *al menos*, Greek *tulachiston*, Hebrew *le-faxot*, Serbian/Croatian *barem*, Czech *aspon*, Japanese *sukunaku-to-mo*

But these authors also observe many languages in which different expressions specialize for just one of the two readings:

- (3) German *mindestens* (epist.) vs. *wenigstens* (conc.) (Gast 2011, Grosz 2011a,b)
- a. Er hat **mindestens** drei Äpfel gegessen
he has at-least three apples eaten
'He ate at least [three]_F apples.'
- b. **Wenigstens** konnte bei diesem Zugunglück [einige]_F Leute gerettet be
at-least could in this trainwreck some people saved be
'At least in that trainwreck, [some]_F people could be saved.'

- cf. also English *at the very least* (epist.), Dutch *mintens, op z'n minst* (epist.), Japanese *-dake-demo* (conc.), Polish *najmniej* (epist.) vs. *chociaz* (conc.)

The challenge: to capture the close kinship between the epistemic and concessive readings in (1a) and (1b), while still leaving room for lexical differentiation.

- what (if anything) is the common semantic/pragmatic core that underlies both readings of *at least*?
- do the two readings arise from a single *at least*, or is this an instance of lexical ambiguity/polysemy?
- why should an expression like *at least* so reliably give rise to one or both of these readings?

2 Preliminary observations

2.1 A more precise characterization of the two readings

Under its epistemic reading, *at least* functions as one of Horn's (1972) implicature-suspenders: by conveying the speaker's ignorance regarding higher scalar values, it calls off the scalar upper-bounding inferences that typically arise in its absence.

- (4) a. Paws ate some of his food.
'The speaker is certain that the higher scalar values (*most*, *all*) are false.'
(scalar upper-bounding inference)
- b. Paws ate **at least** [some]_F of his food.
'The speaker is uncertain of the truth/falsity of the higher scalar values.'
(ignorance inferences)

Under its concessive reading, *at least* generally invites such an upper-bounding inference, which forms part of its overall "settle-for-less" effect:

- (5) **At least** Paws ate [some]_F of his food.
- (i) 'Higher values along *at least*'s associated scale are to be preferred over lower values.' (bouletic inference)
- (ii) 'There are scalar values outranking the stated one.'
(non-maximality inference)
- (iii) 'The speaker is certain that the higher scalar values are false.'
(scalar upper-bounding inference)

Concessive *at least* also conveys that the stated scalar value merits some degree of positive evaluation (e.g., that Paws' eating none of his food would have been the worst possible outcome), yielding a "better-than-nothing" effect.

2.2 At least as a scalar focus operator

Under either its epistemic (6) or its concessive (7) reading, *at least* is able to operate over a diverse range of scales (Kay 1992, Krifka 1999, Geurts & Nouwen 2007):

- (6) a. Paws ate **at least** [some]_F of his food. (epistemic)
 b. Kate won **at least** a [bronze]_F medal.
 c. LeBron scored **at least** [20 points]_F in last night's game.
 d. A: Does Pete like his wife's relatives?
 B: I don't know, but he **at least** likes [her father]_F.

- (7) a. **At least** Paws ate [some]_F of his food. (concessive)
 b. **At least** Kate won a [bronze]_F medal.
 c. **At least** LeBron scored [20 points]_F in last night's game.
 d. A: Does Pete like his wife's relatives?
 B: Not really, but **at least** he likes [her father]_F.

- the two readings are partly distinguished by syntactic position: term-level occurrences of *at least* yield epistemic readings (*pace* Biezma 2013), while for sentence-initial *at least*, the concessive reading is strongly favored (Nakanishi & Rullmann 2009a), if not obligatory (Horn 1972, Kay 1992)

- the morphosyntactic variant *at the very least* appears only to have an epistemic reading, even when it occurs sentence-initially (Kay 1992)

- (8) a. **At the very least**, Paws ate [some]_F of his food. (epistemic)
 b. **At the very least**, Kate won a [bronze]_F medal.
 c. **At the very least**, LeBron scored [20 points]_F in last night's game.
 d. A: Does Pete like his wife's relatives?
 B: I don't know, but **at the very least**, he likes [her father]_F.
 B': #Not really, but **at the very least**, he likes [her father]_F.

For both readings, focus placement has a constraining effect on scale choice and overall interpretation:

- (9) a. **At the very least**, LeBron scored [20 points]_F in last night's game. (epist.)
 b. **At the very least**, [LeBron]_F scored 20 points in last night's game.
 c. **At the very least**, LeBron scored 20 points in [last night]_F's game.
- (10) a. **At least** LeBron scored [20 points]_F in last night's game. (conc.)
 b. **At least**(,) [LeBron]_F scored 20 points in last night's game.
 c. **At least** LeBron scored 20 points in [last night]_F's game.

Simplifying assumption: in the input to semantic interpretation, *at least* attaches to a proposition-denoting constituent, some portion of which bears F-marking.

- (11) at (the very) least [LeBron scored [20 points]_F in last night's game]

- the semantic effect of F-marking, $[\cdot]_F$, is to evoke a set of alternative semantic values, or focus alternatives (Rooth 1985, Fox & Katzir 2011)
- at least* presupposes that these alternatives form partially-ordered scales (compare to *only*, *even* and other scalar focus operators)

- (12) a. $\llbracket \text{Paws ate [some]}_F \text{ of his food} \rrbracket_f = \{ \llbracket \text{Paws ate all of his food} \rrbracket > \llbracket \text{Paws ate most of his food} \rrbracket > \llbracket \text{Paws ate some of his food} \rrbracket \}$
- b. $\llbracket \text{Kate won a [silver]}_F \text{ medal} \rrbracket_f = \{ \llbracket \text{Kate won a gold medal} \rrbracket > \llbracket \text{Kate won a silver medal} \rrbracket > \llbracket \text{Kate won a bronze medal} \rrbracket \}$
- c. $\llbracket \text{LeBron scored [20 points]}_F \text{ in last night's game} \rrbracket_f = \{ \dots \llbracket \text{LeBron scored 22 points in last night's game} \rrbracket > \llbracket \text{LeBron scored 21 points in last night's game} \rrbracket > \llbracket \text{LeBron scored 20 points in last night's game} \rrbracket > \llbracket \text{LeBron scored 19 points in last night's game} \rrbracket > \llbracket \text{LeBron scored 18 points in last night's game} \rrbracket > \dots \}$

3 Epistemic at least in a nutshell

3.1 Epistemic at least as n-ary disjunction: Truth-conditional vacuity and weakening

With entailment scales, epistemic *at least* appears to have no effect on the overall truth conditional meaning:

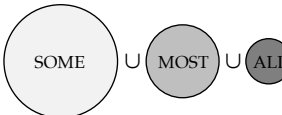
- (13) a. Paws ate some of his food. • (13a,b) compatible with P's eating most/all of food
 b. Paws ate at least [some]_F of his food.

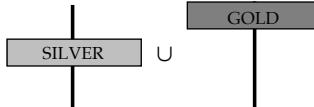
With non-entailment scales, its presence has a weakening effect:

- (14) a. Kate won a silver medal. • only (14b) compatible with K's winning gold
 b. Kate won at least a [silver]_F medal.

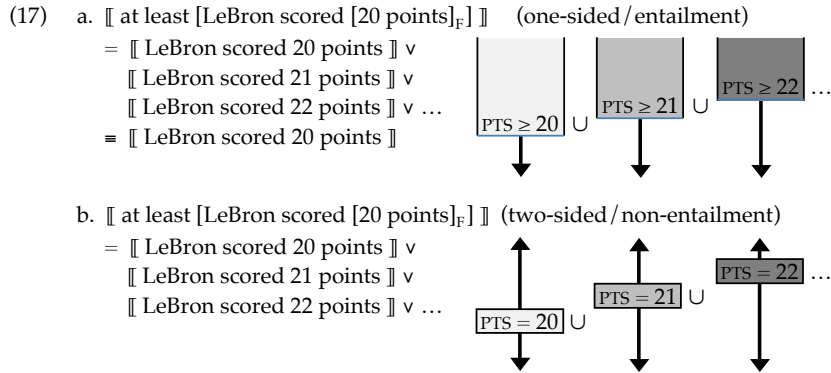
Krifka (1999): *at least* creates an *n*-ary disjunction over its prejacent, or scope, and all higher focus alternatives.

- when the alternatives are ordered by logical entailment, the entire disjunction is truth-conditionally equivalent to its weakest disjunct, i.e., the prejacent itself

- (15) $\llbracket \text{at least [Paws ate [some]}_F \text{ of his food} \rrbracket = \llbracket \text{Paws ate some of his food} \rrbracket \vee \llbracket \text{Paws ate most of his food} \rrbracket \vee \llbracket \text{Paws ate all of his food} \rrbracket = \llbracket \text{Paws ate some of his food} \rrbracket$
- 

- (16) $\llbracket \text{at least [Kate won a [silver]}_F \text{ medal} \rrbracket = \llbracket \text{Kate won a silver medal} \rrbracket \vee \llbracket \text{Kate won a gold medal} \rrbracket$
- 

One-sided/entailment (Horn 1972) vs. two-sided/non-entailment (Horn 1992, Geurts 2006, Kennedy 2013) semantics for bare numerals:



(18) $\llbracket \text{at least } S \rrbracket = \lambda w. \exists q [q \in \{ p \mid p \in \llbracket S \rrbracket_f \ \& \ p \geq \llbracket S \rrbracket \} \ \& \ q(w)]$

Krifka observes, but does not explicitly account for, the ignorance inferences associated with epistemic *at least*:

- early semantic approaches took these inferences to form part of truth-conditional meaning (Geurts & Nouwen 2007, Nouwen 2010)
- much subsequent work has explored their derivation as Quantity implicatures (e.g., Büring 2008, Cummins & Katsos 2010, Mayr 2013, Schwarz 2013, 2016, Ander-Mendia 2015, Kennedy 2015, Nouwen 2015; see also Coppock & Brockhagen 2013 for a Quality-based account, and Cohen & Krifka 2014 for a rather different pragmatic account in terms of meta-speech acts)
- no ignorance inferences arise when *at least* occurs in non-upward entailing contexts, which follows straightforwardly under a Quantity-based pragmatic approach

- (19) a. Did Paws eat at least [some]_F of his food?
 b. If you share at least [two]_F of these ten habits with your partner, your relationship will last forever.
 c. Admission to the event will be free for everyone who donates at least [one]_F canned item to our community food pantry.

3.2 Epistemic at least as *n*-ary disjunction: Symmetric competitors and ignorance

The derivation of both ignorance inferences and scalar upper-bounding inferences is driven by members of a formally-defined set of pragmatic competitors, COMP.

(20) Grover had tuna or chicken for dinner.

COMP(20) = $\{ \underline{T \vee C}, T, C, T \ \& \ C \}$

Upper-bounding inferences within the neo-Gricean Theory of implicature computation (Grice 1967/1975, 1978, Horn 1972, Sauerland 2004, Geurts 2011):

- (i) $\text{BEL}(T \vee C)$ (Quality)
 (ii) $\neg \text{BEL}(T \ \& \ C),$
 $\neg \text{BEL}(T), \neg \text{BEL}(C),$ (Weak Quantity)
 (iii) $\text{BEL}(T \ \& \ C) \vee \text{BEL}(\neg(T \ \& \ C))$ (Competence re: (T & C))
 (iv) $\text{BEL}(\neg(T \ \& \ C))$ (Strong Quantity / Upper-bounding)

Why aren't upper-bounding inferences derived regarding the individual disjuncts?

$\text{BEL}(\neg T), \text{BEL}(\neg C)$ • inconsistent with $\text{BEL}(T \vee C)$ in (i)!

- the individual disjuncts T, C form a pair of **symmetric stronger competitors**: simultaneous strengthening of their Weak Quantity implicatures results in an inconsistent set of inferences
- standard neo-Gricean reasoning derives ignorance inferences for any pair of symmetric competitors (Sauerland 2004, Fox 2007, Schwarz 2013, 2016)

- (v) $\text{POSS}(T) \ \& \ \text{POSS}(\neg T)$ (Ignorance re: the individual disjuncts)
 $\text{POSS}(C) \ \& \ \text{POSS}(\neg C)$

Generalizing from binary to *n*-ary disjunction: COMP contains each individual disjunct and is closed under disjunction, yielding a full set of stronger “quantificational-domain” competitors (Alonso-Ovalle 2006, Chierchia 2013).¹

(21) Grover ate tuna, chicken or duck for dinner.

COMP(21) = $\{ \underline{T \vee C \vee D},$
 $T \vee C, D,$ symmetric
 $T \vee D, C,$ symmetric
 $C \vee D, T \}$ symmetric
 $= \{ \lambda w. \exists p [p \in Q \ \& \ p(w)] \mid Q \subseteq \{ T, C, D \} \}$

- $\text{POSS}(T) \ \& \ \text{POSS}(\neg T)$
 $\text{POSS}(C) \ \& \ \text{POSS}(\neg C)$ (Ignorance re: each individual disjunct)
 $\text{POSS}(D) \ \& \ \text{POSS}(\neg D)$

- reasoning about stronger quantificational-domain competitors has also been called upon to account for the pragmatic behavior of “epistemic” indefinites like German *irgendein* (Kratzer & Shimoyama 2002), Spanish *algún* (Alonso-Ovalle & Menendez-Benito 2010), and Romanian *vreun* (Falaus 2014)

¹ I am omitting any possible competitors to (21) formed through conjunction over the individual disjuncts.

The simple(st?) extension to epistemic *at least*: COMP contains the prejacent and all higher focus alternatives (the individual “disjuncts”), and is closed under disjunction, again yielding a full set of stronger quantificational-domain competitors.

(22) at least [LeBron scored [20 points]_F] (two-sided / non-entailment)

COMP(22) = { $\text{PTS} = 20 \vee \text{PTS} = 21 \vee \text{PTS} = 22 \vee \text{PTS} = 23 \vee \dots$,
 $\equiv \text{PTS} \geq 20$
 $\text{PTS} = 21 \vee \text{PTS} = 22 \vee \text{PTS} = 23 \vee \dots$, $\text{PTS} = 20$, symmetric
 $\equiv \text{PTS} \geq 21$
 $\text{PTS} = 20 \vee \text{PTS} = 22 \vee \text{PTS} = 23 \vee \dots$, $\text{PTS} = 21$, symmetric
 $\equiv \text{PTS} = 20 \vee \text{PTS} \geq 22$
 $\text{PTS} = 20 \vee \text{PTS} = 21 \vee \text{PTS} = 23 \vee \dots$, $\text{PTS} = 22$, symmetric
 $\equiv \text{PTS} = 20 \vee \text{PTS} = 21 \vee \text{PTS} \geq 23$
 \dots }

POSS(PTS = 20) & POSS(¬PTS = 20)
 POSS(PTS = 21) & POSS(¬PTS = 21)
 POSS(PTS = 22) & POSS(¬PTS = 22)
 ...

$\forall n [n \geq 20 \rightarrow (\text{POSS}(\text{PTS} = n) \& \text{POSS}(\neg \text{PTS} = n))]$
 (Ignorance re: prejacent and all higher focus alternatives)

3.3 Outstanding problems and further refinements

Problem #1: unlike ordinary *n*-ary disjunction, epistemic *at least* does not generally convey total ignorance regarding each of its “disjuncts” (Schwarz 2013, 2016, Alexandropoulou et al. 2015, Ander-Mendia 2015, Nouwen 2015).

(23) Paws ate tuna, chicken, or duck for dinner, though...
 #...I’m sure that he didn’t eat tuna.
 #...I’m sure that he didn’t eat chicken.
 #...I’m sure that he didn’t eat duck.

(24) LeBron scored at least [20 points]_F last night, though...
 #...I’m sure that he didn’t score (just) twenty points.
 ...I’m sure that he didn’t score a hundred points.
 #...I’m sure that he didn’t score any more than twenty points.

Epistemic *at least* typically conveys something weaker, namely ignorance about its prejacent and the disjunction of all higher focus alternatives:

(25) at least [LeBron scored [20 points]_F] (two-sided / non-entailment)

POSS(PTS = 20) & POSS(¬PTS = 20) (Ignorance re: prejacent)
 POSS(PTS ≥ 21) & POSS(¬PTS ≥ 21) (Ignorance re: disjunction of all higher focus alternatives)

A possible solution: assume that the competitor set evoked by epistemic *at least* is more impoverished, and thus exhibits less symmetry, than that evoked by ordinary disjunction (Büring 2008, Schwarz 2013, 2016, Kennedy 2015, Nouwen 2015).

COMP(25) = { $\text{PTS} \geq 20$,
 $\text{PTS} \geq 21$, $\text{PTS} = 20$, symmetric
 (\dots) }

Another possible solution: locate the difference between epistemic *at least* and ordinary disjunction not in the definition of their respective competitor sets, but in the determination of their respective quantificational domains (Alrenga 2017).

- Rooth (1992, 1996): scalar focus operators are anaphoric to, and operate over, a contextually-relevant scale *C*, the identity of which is constrained by focus

(26) a. (Uttered during Pete’s trip from Boston to San Francisco vs. the return trip)
 At the very least, he’s made it to [Chicago]_F by now.
 b. A: Does Pete like his wife’s relatives?
 B: I don’t know, but he at least likes [her father]_F.
 (may implicate ‘and possibly her mother too’)
 (will not implicate ‘and possibly Donald Trump too’)

Contextual indeterminacy and privileged prejacent: what is the most that a listener can infer when s/he does not know the speaker’s intended value for the scale *C*?

- the speaker’s decision to evoke *C* with the prejacent $\text{PTS} = 20$ indicates the prejacent’s contextual relevance (= membership in *C*)
- no individual higher focus alternative is guaranteed to belong to *C*, but for the presence of *at least* to be non-trivial, some higher alternative must belong to *C*
- in the extreme case, the most that a listener can infer is what follows under any admissible value for *C*:

POSS(PTS = 20) & POSS(¬PTS = 20) (Ignorance re: prejacent)
 POSS(PTS ≥ 21) & POSS(¬PTS ≥ 21)

Problem #2: epistemic *at least*’s capacity to suspend upper-bounding inferences when it operates over entailment scales goes unaccounted for.

(27) a. Paws ate some of his food.
 b. Paws ate at least [some]_F of his food.

Rather perversely, the competitor sets for (27a) and (27b) turn out to be equivalent!

COMP(27a) = { SOME , MOST , ALL }

BEL(¬MOST) (Strong Quantity / Upper-bounding)
 BEL(¬ALL)

COMP(27b) = { SOME v MOST v ALL ,
 = SOME
 SOME v MOST , ALL , not symmetric !!!
 = SOME
 SOME v ALL , MOST , not symmetric !!!
 = SOME
 MOST v ALL , SOME } not symmetric !!!
 = MOST
 = { SOME , MOST , ALL }

BEL(¬MOST) (Strong Quantity / Upper-bounding !!!)
 BEL(¬ALL)

Possible solution: epistemic *at least* never actually operates over entailment scales.

- in all such putatives instances, a covert scalar focus operator *exh* occurs somewhere in the prejacent and associates with the same F-marked constituent as does *at least* (see Krifka 1991, Wold 1996 on the compositional interpretation of such structures; see Chierchia et al. 2011 on *exh* in *some or all* disjunctions)

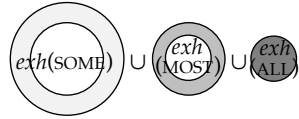
(28) at least [*exh* [Paws ate [some]_F of his food]]

- exh* strengthens the meaning of its prejacent, resulting in an upper-bounded truth-conditional meaning (see Fox 2007 for a more refined version)

(29) $\llbracket exh S \rrbracket = \lambda w. \llbracket S \rrbracket(w) \ \& \ \forall q [q \in \{ p \mid p \in \llbracket S \rrbracket_f \ \& \ p(w) \} \rightarrow \llbracket S \rrbracket \subseteq q]$

- exhaustification of *at least*'s prejacent disrupts any entailments amongst the focus alternatives without affecting the overall truth-conditional meaning

(30) $\llbracket \text{at least } [exh [Paws ate [some]_F \text{ of his food}]] \rrbracket$
 = *exh*(SOME) v *exh*(MOST) v *exh*(ALL)
 = (SOME & ¬MOST) v (MOST & ¬ALL) v ALL
 = SOME



- the presence of *exh* results in symmetric competitors and ignorance inferences

COMP(28) = { exh(SOME) v exh(MOST) v exh(ALL) ,
 = SOME
 exh(MOST) v exh(ALL) , exh(SOME) , symmetric
 = MOST
 exh(SOME) v exh(ALL) , exh(MOST) , symmetric
 exh(SOME) v exh(MOST) , exh(ALL) } symmetric
 = SOME & ¬ALL

POSS(*exh*(SOME)) & POSS(¬*exh*(SOME))
 POSS(*exh*(MOST)) & POSS(¬*exh*(MOST))
 POSS(*exh*(ALL)) & POSS(¬*exh*(ALL))

4 One at least or two?

4.1 Concessive at least as n-ary disjunction? Entailed vs. implicated upper bounds

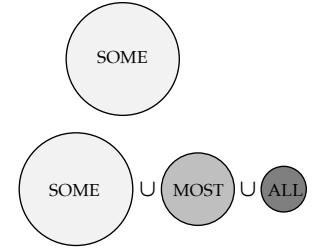
Two different views of concessive *at least*'s truth-conditional import:

- (31) a. Truth-conditionally vacuous (Nakanishi & Rullmann 2009, Grosz 2011)
 $\llbracket \text{at least}_\emptyset S \rrbracket = \llbracket S \rrbracket$
 b. Disjunctive/existential, identical to epistemic at least (a la Biezma 2013)
 $\llbracket \text{at least}_\exists S \rrbracket = \lambda w. \exists q [q \in \{ p \mid p \in \llbracket S \rrbracket_f \ \& \ p \geq \llbracket S \rrbracket \} \ \& \ q(w)]$

Under either view, the inference that higher scalar values do not obtain must (at least) sometimes arise as a (strong) Quantity implicature, namely when *at least* operates over an entailment scale:

(32) $\llbracket \text{at least}_\emptyset [Paws ate [some]_F \text{ of his food}] \rrbracket$
 = $\llbracket Paws ate some of his food \rrbracket$

(33) $\llbracket \text{at least}_\exists [Paws ate [some]_F \text{ of his food}] \rrbracket$
 = $\llbracket Paws ate some of his food \rrbracket \vee$
 $\llbracket Paws ate most of his food \rrbracket \vee$
 $\llbracket Paws ate all of his food \rrbracket$
 = $\llbracket Paws ate some of his food \rrbracket$



Assumption: the set of pragmatic competitors COMP evoked by concessive *at least* consists of (just) the “scalar” competitors derived via substitution for *at least*'s associated scalar term.

COMP(32) = { SOME , MOST , ALL }

COMP(33) = { SOME v MOST v ALL , MOST v ALL , ALL }
 = SOME = MOST
 = { SOME , MOST , ALL }

BEL(¬MOST) (Strong Quantity / Upper-bounding)
 BEL(¬ALL)

The pragmatic competitors evoked by concessive *at least* parallel what is standardly assumed for weak scalar terms like *some* without *at least*, as does the ensuing pragmatic reasoning:

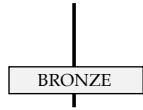
(34) Paws ate some of his food.

COMP(34) = { SOME , MOST , ALL }

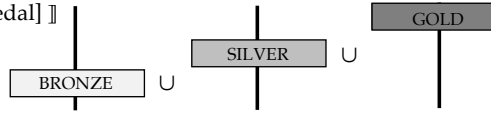
BEL(¬MOST) (Strong Quantity / Upper-bounding)
 BEL(¬ALL)

Question: when concessive *at least* operates over non-entailment scales, is the scalar upper-bounding inference entailed or merely implicated?

(35) $\llbracket \text{at least}_{\emptyset} [\text{Kate won a } [\text{bronze}]_F \text{ medal}] \rrbracket$
 $= \llbracket \text{Kate won a bronze medal} \rrbracket$



(36) $\llbracket \text{at least}_3 [\text{Kate won a } [\text{bronze}]_F \text{ medal}] \rrbracket$
 $= \llbracket \text{Kate won a bronze medal} \rrbracket \vee$
 $\llbracket \text{Kate won a silver medal} \rrbracket \vee$
 $\llbracket \text{Kate won a gold medal} \rrbracket$



COMP(36) = { $B \vee S \vee G$, $S \vee G$, G }

BEL($\neg(S \vee G)$) (Strong Quantity/Upper-bounding)
 BEL($\neg G$)

- the upper-bounding inference may undergo suspension, just like ordinary (strong) Quantity implicatures

(37) a. This hotel may not look very modern, but let's still consider it.
 At least [it's quiet]_F, and it might even turn out to be very comfortable!
 b. You should hire Kate as your research assistant. She's not the best student that I've ever had, but at least she got a [B]_F in my class—maybe even a B+ or an A-.

- response patterns to polar questions with concessive *at least* suggest that the question's truth-conditional content merely specifies a lower bound

(38) A: Will they at least charge him with a [misdemeanor]_F? /
 I am asking whether at least they will charge him with a [misdemeanor]_F.
 B: #No, a felony
 B': Yes, in fact a felony.

(39) A: Will they charge him with a misdemeanor? /
 I am asking whether they will charge him with a misdemeanor.
 B: No, a felony.
 B': #Yes, in fact a felony.

- when concessive *at least* scopes over a possibility modal, the upper-bounding inference that results favors a disjunctive/existential treatment

(40) At least they can (still) charge him with a [misdemeanor]_F.
 'The sp. is certain that they cannot charge him with a felony.'

(41) $\llbracket \text{at least}_3 [\text{they can charge him with a } [\text{misdemeanor}]_F] \rrbracket$
 $= \llbracket \text{they can charge him with a misdemeanor} \rrbracket \vee$
 $\llbracket \text{they can charge him with a felony} \rrbracket$

COMP(41) = { $\diamond M \vee \diamond F$, $\diamond F$ }

BEL($\neg \diamond F$) (Strong Quantity/Upper-bounding)

(42) $\llbracket \text{at least}_{\emptyset} [\text{they can they can charge him with } [\text{misdemeanor}]_F] \rrbracket$
 $= \llbracket \text{they can charge him with a misdemeanor} \rrbracket$

COMP(42) = { $\diamond M$, $\diamond F$ } • no strength relationship...no Quantity implicatures!

Conclusion: concessive *at least* makes the same truth-conditional contribution as does epistemic *at least*—both form an *n*-ary disjunction over their prejacent and higher focus alternatives. The upper-bounding inference that higher scalar values are false appears to pattern uniformly as a strong Quantity implicature.

Such a treatment of concessive *at least* also provides some insight into its non-maximality inference, which plausibly stems from a more general restriction against maximal prejacent to either use of *at least*:

(43) a. #At least Kate won a [gold]_F medal. (sentence-initial, concessive)
 b. #At least Paws ate [all]_F of his food.

(44) a. #Kate won at least a [gold]_F medal. (term-level, epistemic)
 b. #Paws ate at least [all]_F of his food.

- the presence of *at least* in (43),(44) runs afoul of the constraint against locally vacuous semantic composition proposed by Katzir and Singh (2013)

(45) [at least S] is deviant if $\llbracket \text{at least S} \rrbracket = \llbracket S \rrbracket$.

- the constraint in (45) can only be satisfied when there is some focus alternative that is ranked more highly than *at least*'s prejacent
- (with entailment scales, (45) also requires local exhaustive strengthening of *at least*'s prejacent, as discussed in §3.3)

4.2 Is a unified analysis possible? Context vs. conventionalization

Interim summary of the analysis:

Kate won at least a bronze medal. (epist.) vs. *At least Kate won a bronze medal.* (conc.)

$\llbracket \text{at least [K. won a } [\text{bronze}]_F \text{ medal}] \rrbracket$	$\llbracket \text{at least [K. won a } [\text{bronze}]_F \text{ medal}] \rrbracket$
$= \llbracket \text{Kate won a bronze medal} \rrbracket \vee$	$= \llbracket \text{Kate won a bronze medal} \rrbracket \vee$
$\llbracket \text{Kate won a silver medal} \rrbracket \vee$	$\llbracket \text{Kate won a silver medal} \rrbracket \vee$
$\llbracket \text{Kate won a gold medal} \rrbracket$	$\llbracket \text{Kate won a gold medal} \rrbracket$

COMP = { $B \vee S \vee G$,
 $B \vee S$, G , symmetric
 $B \vee G$, S , symmetric
 $S \vee G$, B } symmetric

COMP = { $B \vee S \vee G$,
 $S \vee G$,
 G }

POSS(B) & POSS($\neg B$)
 POSS(S) & POSS($\neg S$) (Ignorance)
 POSS(G) & POSS($\neg G$)

BEL($\neg(S \vee G)$) (Strong Quantity/
 BEL($\neg G$) Upper-bounding)

Perhaps this is all there is to say? Biezma (2013)'s proposed unification:

...for a [concessive reading] to arise it is necessary that (i) the speaker know that no higher alternatives are true ... and (ii) that there be a *valorative* scale (a scale in which higher alternatives are taken to be better). If one of these two requirements is not fulfilled, the [concessive reading] does not arise. With respect to [epistemic readings], they are compatible with a valorative scale, although for an [epistemic reading] to arise it is necessary to assume that higher alternatives may be true. (p. 7)

- when utterance context is consistent with/biased towards speaker's ignorance regarding higher values, pragmatic reasoning over quantificational domain competitors and the ensuing epistemic reading will be possible/favored
- when utterance context is inconsistent with/biased against speaker's ignorance regarding higher values, only pragmatic reasoning over scalar competitors will be possible, resulting in a concessive reading

Question: what about the bouletic inference conveyed by concessive *at least*?

- the inference that higher scalar values are preferable to lower ones arises with concessive *at least* even when the discourse context would strongly bias against such an ordering (Kay 1992, Gast 2011)

- (46) a. At least the big trainwreck caused [some]_F deaths. (concessive)
'More casualties would be a preferable outcome.'
b. (Spoken by an emergency room doctor)
At least he received a [second-degree]_F burn.
'A third-degree (= more severe) burn would be better.'
- (47) a. At the very least, the big trainwreck caused [some]_F deaths. (epistemic)
b. (Spoken by an emergency room doctor)
At the very least, he received a [second-degree]_F burn.

Conclusion: the bouletic inference need not be favored by, or even consistent with, the prior utterance context in order for the concessive reading to arise. Rather, it appears to be part of the conventional import of concessive *at least*.

- the bouletic inference shows the projection behavior of a conventional implicature (or presupposition), persisting unaltered in non-assertive speech acts
- (48) a. Did you at least give him a [second-degree]_F burn? /
I am asking whether at least you gave him a [second-degree]_F burn.
'A third-degree (= more severe) burn would be better.'
b. At least give him a [second-degree]_F burn!
'A third-degree (= more severe) burn would be better.'

Similar observations have been made regarding the ignorance inferences triggered by epistemic *at least* (Geurts & Nouwen 2007, Nouwen 2010, 2015):

- (49) a. A: I have two children. How many children do you have?
B: At least three.
'B is uncertain whether s/he has 4 or 5 or 6 ... children.'
b. #I know exactly how many students are in my class, and it's at least 20.
'The speaker is uncertain whether there are 21 or 22 or 23 ... students.'
- compare to *more than*, which is compatible with ignorance inferences, but does not appear to trigger them in the same way
- (50) a. A: I have two children. How many children do you have?
B: More than two.
b. I know exactly how many students are in my class, and it's more than 20.

Conclusion: the ignorance inference need not be favored by, or even consistent with, the prior utterance context in order for the epistemic reading to arise. Rather, epistemic *at least* appears to be conventionally specified to (obligatorily) evoke its quantificational-domain competitors, which result in ignorance inferences.

- on this sort of "grammaticization" of Quantity implicatures via the obligatorily evoking of pragmatic competitors, see, e.g., Chierchia 2013 and much other recent work on epistemic indefinites

4.3 A note on the "better-than-nothing" effect of concessive *at least*

Recall that apart from its "settle for less" effect, concessive *at least* also conveys that the stated scalar value merits some degree of positive evaluation:

- (51) At least Paws ate [some]_F of his food.
'Paws' eating none of his food is the worst possible outcome.'

A possible solution: concessive *at least* requires that its prejacent not be the lowest-ranked member of its scale (Nakanishi & Rullmann 2009).

- (52) (Spoken of an 8-person race) #At least Kate finished in [eighth place]_F.

Question: why is there a contrast between (52) and (53)?

- (53) (Spoken of an 8-person race) At least Kate won a [bronze]_F medal.

- perhaps (53) is interpreted against a contextually-extended scale that contains the required lower value, e.g., NOTHING < BRONZE < SILVER < GOLD ?

Another possible solution: as part of its bouletic inference, concessive *at least* further conveys that all of the members of its prejacent's scale receive some degree of positive evaluation.

- in (53), *at least* may operate over the lexically-conventionalized scale BRONZE < SILVER < GOLD, since the context still provides a worst possible outcome, viz., not attaining any value along the scale at all
- in (52), the discourse context does not readily provide any worst possible outcome, making it less plausible that the lowest scalar value EIGHTH is to be evaluated positively

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